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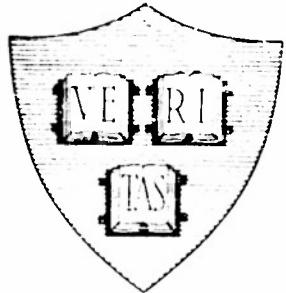
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*1954*

Cruft Laboratory  
Harvard University  
Cambridge, Massachusetts

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PROGRESS REPORT NO. 33



COVERING PERIOD  
JULY 1, 1954 - OCTOBER 1, 1954

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Contract N50RI-76  
Task Orders 1 and 28

October 1, 1954

**Cruft Laboratory  
Harvard University  
Cambridge, Massachusetts**

**Progress Report No. 33**

**Covering Period**

**July 1, 1954 - October 1, 1954**

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**Task Order 1  
NR-071-012**

**Submitted by  
The Steering Committee**

**Task Order 28  
NR-071-011**

**Submitted by  
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**Air Force Contract  
AF19(604)-786**

**Submitted by  
R. W. P. King**

**Air Force Contract  
AF19(604)-1084**

**Submitted by  
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**October 1, 1954**

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171 "Antennas of Discontinuous Radius," C. E. Falick, September 1954.

172 "The Radiation of a Hertzian Dipole over a Coated Conductor," D. B. Brick, May 10, 1953.

173 "A Tabulation of the Fresnel Integrals," Robert D. Turner and Anne F. Downey, March 15, 1953.

174 "End-Correction for Coaxial Line When Driving an Antenna over a Ground Screen," Ronald King, June 15, 1953.

176 "Tandem Slit Diffraction Measurements," L. R. Alldredge, May 18, 1953.

177 "Image-Plane and Coaxial Line Measuring Equipment at 600 MC," Howard W. Andrews, July 1, 1953.

178 "The Collinear Antenna Array: Theory and Measurements," Howard W. Andrews, July 15, 1953.

179 "Mode Control and Operating Voltages of Interdigital Magnetrons," Amarjit Singh, May 5, 1953.

180 "Modification of Standard Network Synthesis Techniques to Use Lossy Elements," J. E. Storer, June 20, 1953.

181 "Injection and Diffusion of Holes and Electrons in a Semiconductor," Harvey Brooks, June 3, 1953.

182 "Full-Wave Detection of Signals in Noise," Noel Stone and David Middleton, July 1, 1953.

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190 "The Polarization of Radio Waves Reflected from the Ionosphere At Nonvertical Incidence, I. Theoretical Background," George Inouye, September 1, 1953.

191 "The Polarization of Radio Waves Reflected from the Ionosphere at Nonvertical Incidence, II. Measurement Techniques and Experimental Results," George Inouye, September 1, 1953.

192 "The Maximum-Minimum Shift Method for Measuring Complex Dielectric Constants and Permeabilities," Ronald King, December 15, 1953.

193 "Folded Antennas," Charles W. Harrison, Jr., December 15, 1953.

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197 "The Nuclear Quadrupole Resonance and Crystal Structure of Solid Iodine," K. W. H. Stevens, April 20, 1954.

198 "Some Spin Wave Properties of Ferrimagnetic and Antiferromagnetic Simple Cubic Crystals," J. S. Kouvel and H. Brooks, May 20, 1954.

199 "Nuclear Magnetic Resonance in Imperfect Crystals," N. Bloembergen, June 18, 1954.

200 "Discontinuities in Open-Wire Lines, Ronald King, July 10, 1954.

201 "The Distribution of Space Charge in the Hull Magnetron Diode," J. A. Bradshaw, August 15, 1954.

202 "Experimental Study of Collinear Slot Antenna," (An Application of Babinet's Principle), Thaddeus Kaliszewski, September 6, 1954.

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I

ELECTROMAGNETIC RADIATION, MICROWAVE CIRCUITS  
AND RANDOM PROCESSES \*

Academic Staff: Prof. R. W. P. King  
Assoc. Prof. C. L. Hogan  
Asst. Prof. R. D. Kodis  
Asst. Prof. J. E. Storer  
Dr. R. V. Row  
Dr. G. Kent

I-A Antennas

I-A-1 Experimental Study of Circular and Square Loop Antennas,  
T. Kaliszewski.

Objective To study the transverse electric field distributions on complementary slot circular and square loops at 10 cm.

Practical Significance The application of Babinet's principle permits the determination of the distributions of current in practically important wire antennas.

Status Measurements of amplitude and phase on circular and square loops have been made for the length of perimeter equal to  $.5\lambda$ ,  $1.0\lambda$ ,  $1.5\lambda$ ,  $2.0\lambda$  and  $2.5\lambda$ . It has been found that the current and phase distribution on both circular and square loops is essentially that of a uniform, short-circuited transmission line with a somewhat increased electrical length. In the case of the square loop this increase is greater when charge maxima are at the corner, less when current maxima are at the corners. A technical report is being prepared in cooperation with Miss P. Kennedy.

I-A-2 Diffraction by Dielectric Materials, T. T. Wu.

Objective To investigate the general problem of diffraction of arbitrary waves by dielectric materials, especially in the form of closed envelopes.

Practical Significance This kind of diffraction is involved whenever a radiating system is near to or enclosed by a dielectric structure.

Status The theoretical study mentioned in the previous report is being continued.

\*Project 1 unless otherwise indicated.

**I-A-3 Propagation of Electromagnetic Waves into Conducting Dielectrics, A. Jayne.**

**Objective** To study the propagation of electromagnetic waves into conducting dielectrics and to study the circuit and field properties of antennas for transmission into conducting dielectrics.

**Practical Significance** Electromagnetic prospecting, underground and under-water radiating systems and the transmission of radio signals into the upper atmosphere are all practical problems which require a knowledge of the propagation of electromagnetic waves into conducting dielectric media.

**Status** For use as the imperfectly conducting dielectric medium in the experimental measurements a liquid mixture exhibiting the properties of relatively high dielectric constant and relatively low loss has been found. This liquid mixture consists of a combination of nitromethane, tetrachloroethylene, and trichloroethylene. By varying the concentrations it is possible to obtain dielectric constants from 15 to 25 and loss tangents from 0.05 to 0.10 at 10 cm wavelength and at room temperature.

Concurrently, the design of the tank and associated probing equipment for making the experimental propagation measurements is nearing completion. Of necessity, the tank will be enclosed by a hood while taking measurements because of the volatility of the liquid mixture.

**I-A-4 Antennas in and over Imperfect Dielectrics, S. Stein.**

**Objective** To study the properties of antennas when immersed in or placed near conducting dielectrics.

**Practical Significance** Propagation over the earth, into the earth, and from one underground location to another, as well as the exploration of the earth by electromagnetic methods, requires a knowledge of the circuit and field properties of antennas near and in the earth.

**Status** Computations discussed in Progress Report 28 are continuing. No graphical results are as yet available.

**I-A-5 Control of Surface Currents, C. E. Faflick.**

**Objective** To study the properties of a lumped impedance section in linear antennas.

**Practical Significance** A lumped impedance, such as a coaxial sleeve, placed along an antenna may be useful in controlling the surface currents to maintain a desired input impedance and field pattern. A knowledge of the control of surface currents is also useful in suppressing undesired currents on feed cables and in reducing scattering cross sections.

**Status** The experimental aspect of this investigation has been completed. Technical reports 157 and 171 are nearing completion.

**I-A-6 Coupled Receiving Antennas, A. Vobach , C. Moritz.**

**Objective** A study of the behavior of the two-element array.

**Practical Significance** Applications of the study of coupled receiving antennas are to direction-finding, directional receiving arrays, and by application of the reciprocal theorem, to directional transmitting arrays.

**Status** Computations from the measurement data which have been obtained and reported in Progress Report No. 28 are continuing.

**I-A-10 Antenna Measurements on the Image-Plane Line, P. Kennedy.**

**Objective** The research in this project is directed toward the general study of various types of antennas driven by an image-plane line.

**Practical Significance** In order to provide complete data on the behavior of various types of antennas not applicable to coaxial line techniques and to verify theoretical studies, experimental measurements are required.

**Status** Impedance measurements have been made for a set of circular loops of circumference  $\lambda/2$  to  $2\lambda$ . The results have been plotted as a function of half-circumference and appear to agree very well with theoretical values (See I-A-14). End-effect and spacing corrections will improve the agreement slightly. A report is in progress.

**I-A-13 The Square-Loop Antenna, R. W. P. King.**

**Objective** The determination of the distribution of current and the impedance of a square loop antenna as a function of its dimension and wire size. The loop may be driven at one or more corners or at the center of one or more sides; it may be loaded at one or more corners or at the centers of one or more sides.

**Practical Significance** No solution of the circuit properties of square loops of dimension comparable with the wavelength is available. Such loops have applications in direction-finders, beacons, etc.

**Status** The theoretical study of the square loop antenna driven at each corner is continuing. The two phase sequence for which solutions for the current are sought have been reduced to a single

integral equation the formal solution of which is in progress.

I-A-14 The Circular Loop Antenna, J. E. Storer.

Objective	To determine the current distribution and impedance of circular loop antennas.
Practical Significance	When the loop dimensions are comparable to the wavelength numerical solutions of the loop impedance and current distribution have not been available.
Status	Computations have been completed and a technical report will be issued shortly.

I-A-16 Transient Characteristics of Antennas, R. V. Row.

Objective	To investigate theoretically and experimentally the receiving and transmitting behavior of various antennas when subject to pulse excitation.
Practical Significance	An understanding of the transient behavior of simple antennas will be of value in the practical design of large directive antennas for radar.
Status	A sweep generator for the high-speed micro-oscillograph is under construction. Several leaks in the vacuum system are being repaired.

The construction of a spark-gap transmission-line pulser for generating 20 kv pulses with a rise time of approximately  $5 \cdot 10^{-10}$  seconds is nearing completion. This pulser is described by R. C. Fletcher in Technical Reports 20 and 21 from the Laboratory for Insulation Research at M.I.T.

I-A-17 Single Conductor Lines over Coated Conducting Planes, C. Shafer

Objective	Theoretical and experimental investigation of single conductor lines over conducting planes coated completely or partially with dielectric.
Practical Significance	The recently developed and commercially used microwave-strip circuits involve such lines.
Status	A horizontal ground screen of sufficient flatness for accurate measurements of the fields of microwave strip lines from 20 to 100 centimeter wavelengths has been constructed. This consists of a 8' x 4' x 1/4" dural sheet supported by two longitudinal I-beams and cross-channel beams. This structure is supported at a height of 5 1/2 feet by a substantial wooden frame under which the signal and detector equipment will be located.

The final testing and calibration of the equipment has been postponed in order to continue work on the theoretical aspects of the problem.

### I-B Microwave Optics

#### <sup>†</sup>I-B-4 Back-Scattering Measurements, R. V. Row.

Objective	The research in this project is directed toward the general study of back-scattering.
Practical Significance	A rapid and accurate method for measuring back-scattering from arbitrary obstacles is useful in determining radar cross sections.
Status	At the request of the Air Force Cambridge Research Center, a study has been initiated to determine the feasibility of measuring the back-scattering from conducting prolate spheroids for a nose-on incident field. For a spheroid with a major-to-minor-axis ratio of 10:1 and $2\pi a/\lambda = 0$ , theory indicates a back-scattering cross section of $3.10^{-4} \text{ cm}^2$ at a wavelength of 10 cm. It is proposed to make this study using the recently constructed 24 x 48 ft image plane at a frequency in the range 600 to 800 Mc/s.

#### I-B-5 K-band Diffraction, C. Tang, R. V. Row, R. D. Kodis.

Objective	To investigate diffraction by obstacles whose dimensions are large compared to a wavelength and to extend the present knowledge of the transition from the static to optical regions in scattering problems.
Practical Significance	These problems are of interest in radar back-scattering.
Status	The back-scattered field is measured by the technique employed by Sevick using incident field cancellation by means of a Magic-T. The first series of measurements was not reproducible due to poor contact between the cylindrical disks and parallel plates. Modifications have been made on disks and plates to ensure good contact. It is now found that the back-scattered signal is near the noise level of the receiving system presently used. It is necessary to improve the sensitivity of the system. Methods such as the use of a K-band spectrum-analyzer as a receiver, or the use of a coherent detector are under investigation.

#### I-B-9 Unflanged Semi-infinite Rectangular Guide, M. Balser, R. V. Row.

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<sup>†</sup>Air Force Contract AF 19(604)-786.

**Objective** To investigate the frequency dependence of the dominant-mode reflection coefficient from the open end of an unflanged rectangular guide.

**Practical Significance** An experimental measurement of this quantity is necessary as a check on an approximate theory for a previously unsolved problem.

**Status** No progress is reported in the measurements or computations for this quarter.

<sup>†</sup>I-B-20 Theory of Electromagnetic Corrections to Geometrical Optics,  
R. D. Kodis, I. Stakgold.

**Objective** The development of asymptotic expansions for scattering at high frequencies through the application of variational principles.

**Practical Significance** The successful application of a general method to simple scattering problems at high frequencies may provide valuable insight into more complicated problems.

**Status** A modified variational principle has been used successfully to obtain the first term of an asymptotic expansion for the scattering cross section of an infinite cylinder. The question is now being investigated whether an equivalent result can be obtained with the same trial function from the conventional variational principle.

The earlier examination of the Kirchhoff approximation for scattering problems has led to an attempt to correct the various approximations that have been developed: geometrical optics, physical optics, variational. This work will be summarized in a separate report.

<sup>†</sup>I-B-21 The Scattering of Spherical Waves by Cylinders, D. B. Brick.

**Objective** The theoretical study of the current distributions excited by a spherical wave on cylindrical scatterers of arbitrary cross section.

**Practical Significance** The relationship between plane-wave scattering and spherical wave scattering is useful in relating experimental data to the theoretical results for cylindrical configurations.

**Status** The theoretical investigation of the scattering of the field of an arbitrarily oriented dipole by a circular cylinder has been completed. The study of the more general problem of scattering by a cylinder of arbitrary cross section is well under way. The results already obtained indicate that some very interesting and fundamental relationships exist between plane-wave and spherical-wave scattering.

**I-B-22 Theoretical and Numerical Investigations of Optical Current Distributions, A. Vobach, D. B. Brick.**

**Objective** To investigate numerically the current distributions on cylindrical scatterers using classical and quasi-optical means.

**Practical Significance** A knowledge of the behavior of the current distributions on scatterers as their characteristic dimensions and radii of curvature vary is a prerequisite to the extension of the optical and quasi-optical scattering approximations to lower frequencies.

**Status** The series solutions for the current distributions of elliptic cylindrical scatterers have been obtained. Steps are now being taken to obtain tables of Mathieu over a larger range of values than are presently available in order to compute the current distributions on cylinders for which the quasi-optical approximations may be expected to be valid approximations. (The formulations of the quasi-optical current distributions are also under way).

**I-B-23 Aperture Scattering at High Frequencies, H. Chang.**

**Objective** To investigate the diffraction at high frequencies of a plane electromagnetic wave by a circular aperture in an infinitesimally thin, perfectly conducting plane screen.

**Practical Significance** The successful application of a general method to a simple scattering problem at high frequencies may provide insight into more complicated problems.

**Status** The present research is based on the work of Levine and Schwinger. They have shown that the aperture diffraction problem depends on either the knowledge of the surface current induced on the shadow side of the screen or the tangential electric aperture field. The vector integral equations which they give for these quantities have been solved in the high-frequency region with normally incident plane waves. The transmission coefficient of the aperture (ratio of the energy passing through the aperture per second to that transported per unit area of the incident wave) has been obtained and compares very favorably with the exact values of Meixner and Andrejewski. Asymptotic expressions for the transmission coefficient as a function of  $1/ka$ , where  $k = 2\pi/\lambda$  and  $a$  is the radius of the aperture, have also been obtained. Details of this research will be made available in a forthcoming technical report.

**I-B-24 The Scattering of Plane Waves by Spheres, S. I. Rubinow.**

**Objective** To obtain a theoretical correction to geometrical optics for scattering of plane waves by spheres at high frequencies.

Practical Significance      The knowledge of and method of obtaining such a correction should be useful for the investigation of scattering from arbitrary-shaped objects.

Status      At the present time, a review of significant literature is being undertaken.

**I-B-25 Experimental Investigation of Optical Current Distributions,  
D. B. Brick.**

Objective      To measure the current distributions on large elliptical cylinders of various size and eccentricity.

Practical Significance      The experimental determination of current distributions is necessary for estimating the validity of analytic approximations.

Status      The modification of the X-band ground screen equipment is complete. The apparatus is being tested and calibrated against the known results for a circular cylinder.

**I-C Microwave Circuits**

**††I-C-1 Tensor Permeability and Dielectric Constant of Anisotropic Media,  
C. L. Hogan.**

Objective      To measure, in the frequency range 500-24,000 Mc/s the tensor properties of media which are rendered anisotropic by an impressed magnetic field.

Practical Significance      The design of the antireciprocal elements in microwave gyrators and circulators, one-way transmission systems, etc., depends upon a detailed knowledge of the anisotropic properties of various media under influence of a steady magnetic field.

Status      Experiments are being designed to measure the tensor components of the permeability, and, in particular, the necessary equipment to make measurements at X-band frequencies has been ordered and is being assembled.

**††I-C-2 Antireciprocal Elements in TEM Mode Systems, C. L. Hogan,  
J. Pippin.**

Objective      Theoretical and experimental investigation of the feasibility of building antireciprocal elements in guided wave systems propagating a TEM mode.

Practical Significance      If successful, these experiments will yield data making it possible to build antireciprocal elements in strip lines

**†† Air Force Contract AF19(604)-1084.**

and possibly coaxial lines.

**Status** A sample of indium-antimonide has been obtained from Dr. R. G. Breckenridge of the Bureau of Standards. Preliminary measurements on the sample were not as encouraging as expected and further measurements are waiting the completion of the experimental equipment described in I-C-1. When this equipment is completed it is hoped to measure the complex tensor conductivity of indium antimonide as a function of temperature when it is biased with a d.c. magnetic field.

Several strip line structures have been designed and are now under construction in which it is hoped non-reciprocal effects can be obtained by using ferrites instead of semi-conductors. Tests should be completed on some of these structures within a few weeks and if the preliminary results are sufficiently promising this phase of the program will obtain major emphasis for some time; it not only will allow the construction of all of the presently known non-reciprocal devices in a strip line but it also promises to have tremendous importance relative to the problem of building antireciprocal elements at frequencies below 1000 Mc/s.

#### I-D Electronics

##### I-D-1 Investigation of a Modified Type of Barkhausen-Kurz Oscillator, G. Kent.

**Objective** To investigate the potentialities as a microwave power source of a Barkhausen-Kurz (B-K) tube which is especially designed to provide simple harmonic electron motion and to use efficiently the B-K energy-conversion process.

**Practical Significance** It is believed that a relatively high-efficiency microwave generator with frequency limitations comparable to those of the reflex klystron might be developed.

**Status** There is nothing new to report on this project for the current period.

##### I-D-3 Space-Charge-Wave Oscillations in the Magnetron, J. Osepchuk.

**Objective** The experimental and theoretical study of space-charge waves in the magnetron diode.

**Practical Significance** The existence of space-charge waves in the magnetron diode is significant in considering low-current behaviour in multicavity magnetrons and would indicate the possibility of its use as a microwave signal generator.

**Status** A resonance search on the magnetron diode has been conducted. In the frequency range of 40<sup>0</sup> - 4000 Mc/s

only one prominent resonance was observed. The resonant frequency was 2680 Mc/s and corresponds roughly to the quarter-wave resonance of the slab-line transformer between anode slot and coaxial line. Also at about 2240 Mc/s there was an indication of an output resonance.

#### I-D-4 Electrolytic Tank, P. Kennedy, G. Kent.

**Objective** Design and construction of an electrolytic tank and associated apparatus.

**Practical Significance** The electrolytic tank is useful in solving Laplace's equation for two-dimensional problems or three-dimensional problems with axial symmetry such as those which occur in electron-optical systems and other electron devices.

**Status** Construction of the electrolytic tank has been completed. Some preliminary measurements have been made on soluble problems to obtain a measure of experimental error. A solution of a known concentration of  $\text{NiSO}_4 \cdot 6\text{H}_2\text{O}$  and distilled water was used as the electrolyte, and the comparative merits of chemically cleaned nickel and rhodium-plated electrodes were considered. Effects due to boundary conditions, surface tension and the use of a liquid detergent to counteract it, and nonlinearity resulting from high current density have been investigated.

#### I-D-11 Circuits with Nonlinear Inductance, N. S. Prywes.

**Objective** Investigation of oscillations in circuits with nonlinear inductances.

**Practical Significance** The possibility of constructing frequency multipliers and high efficiency multiple frequency power supplies is indicated. Capacitive loading of magnetic amplifiers is another obvious use.

**Status** A technical report is in preparation.

### I-E Random Processes

#### I-E-2 Special Topics: Optimum Linear Filters for the Integration of Random Waves in Finite Time, J. Storer.

**Objective** To study the conditions under which random waves may be averaged in finite time intervals by means of linear passive networks or filters. In particular, optimum filters for the smoothing of random functions are desired, as well as the conditions needed for such optimum integration.

**Practical Applications** One direct practical application of this study lies in the reception of intelligence in the most efficient way for the always

limited time at the receiver's disposal.

**Status** The analytical features of a technical report have been completed.

**I-E-4 Rectification of Nongaussian Noise, J. A. Mullen.**

**Objective** To obtain the quantities of physical interest when a signal and nongaussian noise are rectified.

**Practical Application** This work will make more types of noise amenable to analytic treatment and will check the sensitivity of the rectified output to the input statistics.

**Status** The output power is being calculated and the analytical work is being extended to include the low density output correlation function, using Rayleigh's and Pearson's approximation methods for the random walk.

**I-E-5 The General Theory of Reception of Signals in Noise, D. Van Meter.**

**Objective** By formulating the reception problem in its most general terms this research seeks to show how detection and extraction of signals may be distinguished operationally in a manner independent of arbitrary criteria of performance, and to indicate what methods of modern statistical analysis are to be used in determining optimum performance of receiving systems.

**Practical Applications** The theory shows what are the best systems for a given purpose, with a definition of "best" consistent with the particular external constraints of the problem; and indicates to what degree a given practical system may depart from the ideal, permitting improvement to be related to cost in a meaningful way.

**Status** A paper entitled "Modern Statistical Approaches to Reception in Communication Theory" by D. Van Meter and D. Middleton was published in the Transactions of the I. R. E. for September and presented at the Symposium on Information Theory at M. I. T. on September 17. Further detailed properties of Bayes and Minimax extractors with results for several specific problems have been obtained. The technical report on this work is now in active preparation.

**I-E-6 Study of Output Spectra of Several Nonlinear Devices when Fed by Narrow-Band Noise Plus an Unmodulated Carrier, G. Fellows.**

**Objective** Experimental verification of Middleton's theory on the subject, and an investigation of the problems of spectral analysis.

Practical Applications The equipment will be used to obtain useful results for special cases of extreme computational difficulty.

Status Final experimental runs are now complete for the case of a half-wave linear detector. The equipment is now being prepared for investigation of one half-wave quadratic detector.

I-E-7 Experimental Investigation of the Statistical Properties of Noise and Signal Waves in FM Receivers, H. Fuller.

Objective To extend, by experimental measurements, the theoretical results of Middleton to cases of non-ideal receiver models that are impractically difficult to treat analytically.

Practical Applications The signal-to-noise ratio at the output of the receiver as a function of the several design parameters of an FM receiver will be determined. Together with the signal spectrum distortion this information should allow a closer approach to an optimum receiver design for a specific purpose.

Status Further work has been done on the analytical determination of the limiting form of the output spectrum of an FM receiver for large  $\Omega$ . A suitable frequency modulator has been designed and breadboard-tested. A wide-band multiplier is being constructed.

I-E-8 Sequential Detection of Signals in Noise, J. J. Bussgang.

Objective To study the application of sequential probability ratio tests to the problems of detection of signals in noise.

Practical Significance A sequential detector requires, on the average, fewer observations than the classical fixed sample-length detector for the same probabilities of errors. It is important to carry out a theoretical analysis of sequential detectors before experimental applications are made.

Status A study of sequential detection has been undertaken. Results obtained will be outlined in a Technical Report (No. 175). The report is in preparation.

## II

## ELECTRON AND SOLID STATE PHYSICS

Academic Staff: Prof. J. H. Van Vleck  
Prof. H. Brooks  
Assoc. Prof. N. Bloembergen  
Dr. F. K. Willenbrock

## II-A Radio and Microwave Spectroscopy of the Solid State

A considerable number of coordinated investigations of nuclear and electronic spin resonance in various types of solids are intended to result in a better understanding of the structure and behavior of magnetic spin systems and of the nature of crystalline imperfections in solids.

## II-A-1 Nuclear Resonance in Metals and Alloys, T. J. Rowland and N. Bloembergen.

Objective To obtain information about electronic structure and diffusion in pure metals and alloys by measuring nuclear resonance line shapes, widths, and shifts as functions of concentration of various solute metals, field strength and isotopic concentration. Quadrupole effects in metals are also being investigated.

Status The experiments on the  $Tl^{203}$  and  $Tl^{205}$  magnetic resonance in thallium and thallic oxide with enriched isotopic compositions have been completed. Definite proof of the existence of exchange and pseudo-dipolar interactions between the nuclear spins has been obtained. A phenomenological and an atomistic theory for these interactions has been developed. Evidence for the anisotropy of the chemical shift in thallic oxide has also been obtained. These results are reported in detail in Technical Report 205, which is in course of publication.

## II-A-2 Nuclear Magnetic Relaxation in Metals, Alloys, and Semiconductors, A. G. Redfield.

Objective To obtain the nuclear magnetic relaxation time in metals, alloys, and semiconductors by measuring the nuclear magnetic resonance line shape as a function of applied radio-frequency power, and to experimentally and theoretically investigate the behavior of nuclear spin systems at high rf power levels.

Status The experimental data on  $Cu^{63}$  and aluminum in the pure annealed metals have been rerun using a lower modulation frequency (18 cycles). The previous results quoted should be accepted with caution since the forty cycle-modulation period was comparable with the spin-lattice relaxation time.

The theory of saturation described in the last report has been developed further by assuming that above saturation the spin system is in its state of highest entropy with respect to the transformed Hamiltonian in the precessing coordinate system (z-axis along the effective field). The effective energy in the precessing system, and thus the magnetization, are determined by the requirement that in the steady state, spin lattice collisions (regarded as a small perturbation) keep the effective energy constant on the average. The entropy of the spin system is difficult to estimate so that the resulting theory is qualitative, but it appears to cover rather generally the behavior of spin systems in large rf fields. A technical report describing this theory and the related experimental data is in preparation.

A cryostat is being constructed to carry on these measurements down to liquid helium temperatures.

#### II-A-3 Nuclear Magnetic Spin-Spin Relaxation, K. Dwight.

**Objective** To measure spin-spin relaxation times in systems of nuclear magnetic dipoles when energy is absorbed at audio-frequencies in weak external magnetic fields.

**Status** The leak in the dewar having been repaired, the dewar was tested with liquid nitrogen. The liquid boiled vigorously, and was gone in 15 minutes. Calculations showed that such excessive heat transfer could be almost completely accounted for by heat radiation, and so we dismantled the dewar and carefully polished its surfaces.

Upon reassembly, the dewar was found to hold liquid nitrogen for about an hour, but the boiling was still too rapid. We then placed the tail of the dewar inside another container filled with liquid nitrogen, the second container being a hollowed-out block of polyfoam with a minimum wall thickness of 1/2 inch. With this arrangement the liquid surface in the dewar was nicely quiescent, and the dewar could hold the liquid for several hours.

An attempt was made to observe the nuclear resonance in the liquid nitrogen, but in spite of the quiescent surface there proved to be too much disturbance of the r-f coil. This disturbance affected the capacitance between the coil and ground, which in turn caused the frequency of oscillation to fluctuate violently over a range of some 40 kilocycles, necessitating an excessively long time-constant for detection of the resonance signal.

The r-f coil was rebuilt out of heavier wire, with special attention to the rigidity of the wires leading into the coil. It was virtually impossible for outside disturbances to shake the coil or its leads, but fluctuations of about 10 kc remained. Calculations showed that this could be due to the bubbling of nitrogen between the coil and its grounded shield, because of the different dielectric constants of the liquid and the gas. This space was filled with beeswax and the fluctuations were reduced to 1 kc, which is quite tolerable when working with  $\text{CaF}_2$ , because of the 40 kc width of its

nuclear resonance.

An attempt to obtain data on the spin-spin relaxation time demonstrated the impossibility of manually controlling the length of time spent in the weak field with sufficient accuracy. Hence a device was constructed to automatically control the transfer mechanism. The time interval can thereby be adjusted by 1/6 sec steps, and is accurate to a fraction of that.

Some preliminary data have been obtained, and indicate the desirability of increasing the power output of the audio system. Changes are being made for this purpose.

**II-A-10 Paramagnetic Resonance in Impurity Semiconductors, F. K. Willenbrock.**

**Objective** To measure the paramagnetic resonance due to the unpaired electron spin of impurity atoms in n- and p-type semiconductors. From this measurement, the g-factor and spin-lattice relaxation time of the electrons and holes can be determined.

**Status** No progress was made on this project during this report period.

**II-A-11 Paramagnetic Resonance in Alkali Metals, R. H. Silsbee.**

**Objective** To study the magnetic resonance of the conduction electrons in colloidal particles of alkali metals in alkali halide crystals.

**Status** Two narrow lines of width 1.5 and 4 gauss with g-values very near the free electron value have been observed in two colored crystals of sodium chloride. Unfortunately the results have not been reproducible from one crystal to another, and the intensity of the absorption is much lower than expected. Optical equipment is now being set up to determine the colloid particle size from the optical absorption bands of the crystal.

**II-A-20 Ferromagnetic Resonance in Single Crystals of Nickel, Cobalt, and Nickel Ferrite, C. J. Hubbard.**

**Objective** To obtain the ferromagnetic resonance line width in single crystals of nickel ferrite ( $\text{NiO} \cdot \text{Fe}_2\text{O}_3$ ) at various temperatures down to 4°K, at frequencies in the 1 cm region. Other substances are to be studied later.

**Status** The ferromagnetic resonance absorption in a .010-inch diameter spherical single crystal of nickel ferrite has been investigated at room temperature. The 16 Mc/s detuning of the cavity caused by the sample is now within the bandwidth of the microwave components. The absorption maxima fell at the same field as for the previous

sample, but the width appeared much narrower, being approximately 2 per cent of the resonant field value. A current-regulator will be necessary to investigate this line with any precision, and one is being built.

## II-B Properties of Electrons in Solids

Several investigations, using widely different experimental techniques, are all concerned with the behavior of electrons in metals and semiconductors. They include the measurement of Hall effect in ferromagnetic materials, electronic spin specific heat, magnetoresistance and properties of semiconductors under hydrostatic pressure.

### II-B-1 Hall Effect in Ferro- and Ferrimagnetics, J. Lavine.

**Objective** To investigate some of the electrical and magnetic properties of ferrites by means of the Hall measurement. The purpose of the measurement is to obtain information about the conductivity mechanism in ferrites.

**Status** Measurements of  $R_1$  vs  $\rho$  for three Ni-Fe alloys were made in the temperature region between liquid air and room temperatures. These alloys were supermalloy, mumetal (Bell Telephone Laboratories), and Carpenter Hymu 80 (Carpenter Steel Co.) with compositions of approximately 80 per cent Ni and 16 per cent Fe. In no case did  $R_1$  possess the  $\rho^2$  dependence required by the Karplus-Luttinger theory. In all cases,  $R_1$  was positive, compared with a negative  $R_1$  in Ni.

An appreciable part of this report period was spent in determining the effects of the change in magnetization over the range of  $H$  in which  $R_o$  is measured, upon the measurements of  $R_o$ . The normalized Hall voltage is given by

$$V_H = R_o H + R_1 M$$

where  $R_o, R_1$  are functions of temperature only.

$$\frac{\Delta V_H}{\Delta H} = R_o + R_1 \frac{\Delta M}{\Delta H} = R_o \left(1 + \frac{R_1}{R_o}\right) \frac{\Delta M}{\Delta H}$$

In the case of Ni where  $R_1/R_o \sim 10^2$ , a change in  $M$  of the order of 1/2 gauss in 5000 oersteds produces a 1 per cent error in  $R_o$ . In the case of  $Fe_3O_4$  where  $R_1/R_o \sim 10^3$ , a similar change in  $M$  produces a 10 per cent error.

The empirical approach to saturation is given by (R. M. Bozorth, Ferromagnetism, D. Van Nostrand, 1951, p. 486)

$$M = M_s \left(1 - \frac{a}{H} \dots\right) + K_o H$$

where  $a$  is the magnetic hardness, and  $K_o$  is due to the increased alignment of spins in the field  $H$ .

$$\frac{\Delta M}{\Delta H} = \frac{M_s a}{\Delta H} \left( \frac{1}{H_1} - \frac{1}{H_2} \right) + K_o = \frac{M_s a}{H_1 H_2} + K_o$$

where

$$\Delta H = H_2 - H_1$$

and

$$H_2 > H_1 \neq 0.$$

Therefore, if  $H_1 H_2 \gg M_s a / K_o$ , we can ignore the effect of the magnetic hardness. However,  $a$  is not readily known since it varies with sample structure and history.

Values of  $a$  and  $K_o$  for Ni given by Bozworth (op. cit.) indicate that  $M$  changes by about 1 gauss in 5000 over the range of  $H$  used in the  $R_o$  measurement. In the case of  $\text{Fe}_3\text{O}_4$ , the value of  $K_o$  calculated from the Holstein and Primakoff (T. Holstein and N. Primakoff, Phys. Rev. 58, 1098, 1940) formulation indicates a change in  $M$  of 1.2 gauss for 5000 oersteds and hence an error of  $\sim 25$  per cent in  $R_o$ . An estimate of the magnetic hardness,  $a$ , in  $\text{Fe}_3\text{O}_4$  by measurement of  $\partial\mu/\partial H$  where

$$\mu = 1 + 4\pi = 1 + 4\pi \frac{\partial M}{\partial H}$$

$$\frac{\partial \mu}{\partial H} = 4\pi \frac{\partial^2 M}{\partial H^2} = \frac{-8\pi M_s a}{H^3}$$

indicates that this contribution to  $\Delta M / \Delta H$  can be ignored in comparison with  $K_o$ .

## II-B-2 Specific Heats of Spin Wave Systems, J. S. Kouvel.

**Objective** To determine the spin wave contributions to the specific heats of ferrimagnetic and antiferromagnetic materials at liquid helium temperatures.

**Status** The construction of a new liquid helium cryostat was completed. Its general design features are: (1) brass and stainless steel construction of the two cylindrical dewar vessels; (2) no "tail" - since the cryostat is to be used for calorimetry work in the absence of a magnetic field; (3) the two concentric dewar vessels are completely separable - i.e., there are no common solder joints; (4) maximum capacities: 1.5 liters of liquid helium in the inner dewar and 5 liters of liquid nitrogen in the outer dewar. Subsequent tests have shown that a vacuum of one micron or less can be maintained in either vacuum jacket for over 24 hours after the valves to the pumps are closed.

The first few attempts to transfer liquid helium from the storage vessel into the cryostat were unsuccessful. The reasons for an extremely

rapid evaporation of the liquid helium in the cryostat during the transfer were traced to very large oscillations of the cold helium gas column in the cryostat. This problem was finally overcome by the insertion of a row of mica baffle plates into the cryostat and by a very slow rate of transfer of the liquid helium. With the first successful transfer, we found the mean evaporation rate of the liquid helium in the cryostat for the initial 8 hours to be less than 30 c. c./hr.

The resistance of the carbon resistance thermometer in the calorimeter was calibrated from 4.2°K down to 1.5°K against the vapor pressure of the liquid helium bath. This was subsequently repeated; the calibration varied insignificantly. In the course of the first calorimetry run (on the empty calorimeter), it was found that there was extraneous interference between the heater timing circuit and the thermometer circuit and that the thermal leaks between the calorimeter and the liquid helium bath were somewhat excessive. However, it appeared that thermal equilibrium could be quickly established throughout the calorimeter. Thus, at least for our preliminary measurements, we could hope to get a fair estimate of the specific heat by measuring the initial slope of the heating curve-using a stop watch to record the time. Ideally (i.e., for perfect thermal equilibrium throughout the calorimeter), the initial  $dT/dt$  should be equal to  $P/C$ , where  $P$  is the heating rate and  $C$  is the specific heat. In this way, measurements between 1.8 and 4.2°K were carried out on the empty calorimeter and on the calorimeter filled with powdered magnetite (in a helium atmosphere). The results definitely show that besides the lattice heat, which at these temperatures may be expected to have a cubic temperature dependence, there is a contribution to the specific heat of magnetite that varies as a much lower power of the temperature. However, the appreciable scatter of our experimental points makes it impossible to deduce its exact temperature dependence, although it appears to be definitely between  $T$  and  $T^{3/2}$ . Interestingly enough, its magnitude seems to be very close to that predicted by the spin wave analysis discussed in Technical Report No. 198. Changes in the experimental system, that should result in improved accuracy of the data, are now in progress.

A first approximation to the spin wave treatment of the inverse spinel structure has been completed. The only exchange interactions considered were those between ions on tetrahedral or A-sites and their nearest neighbors on octohedral or B-sites. The resulting expressions for the spin specific heat and relative magnetization are very similar to those previously obtained for the ferrimagnetic simple cubic structure (T.R. No. 198). This analysis of the inverse spinel is now being extended to include AA and BB exchange interactions.

#### II-B-3 Magnetoresistance in Strained Metals, C. W. Maynard.

Objective            To investigate the change in resistance of metals at the temperature of liquid helium when subjected simultaneously to a magnetic field and to a strain.

Status            The experimental technique for handling liquid helium has

been improved and objectionable pickup and noise in the electrical circuits has been reduced. Further work on the theory of the magneto-elastic resistivity is in progress.

**II-B-4 High-pressure Effects in Semiconductors\*** W. Paul and H. Brooks.

**Objective** To investigate the electric, magnetic and optical properties of semiconductors at pressures up to 30,000 kg/cm<sup>2</sup>.

**Status** Parts of this project are being carried out in collaboration with Group 35, Project Lincoln, at M.I.T. Extensive use is being made of the facilities available in Professor Bridgman's laboratory.

**a. Measurement of Hall Constant and Magnetoresistance\*\***

Measurements of the Hall effect, electrical resistance, longitudinal and transverse magneto-resistance have been made as a function of pressure, in the range from 1-10,000 kg/cm<sup>2</sup>, at 0°C, on germanium n- and p-type samples oriented in both 100 and 110 directions. In the magneto-resistance measurements it was observed that the voltage drop across the resistance-determining probes was not invariant in magnitude under reversal of the direction of the magnetic field. Analysis of the data is in progress, taking into account the recent determination of the energy band structure of germanium through cyclotron resonance experiments. Tests are also planned to reexamine the anomalous behaviour of the voltage across the resistance probes on field reversal.

Tests of the suitability of Be-Cu for an externally unsupported pressure vessel in the range up to 25,000 kg/cm have been satisfactorily completed. The bomb is now being constructed.

**b. Measurement of the Optical Properties\*\*\***

In the last report we indicated that the light transmission through sapphire pressure-bomb windows could be made almost completely independent of pressure by "seasoning" the windows with several cycles of applied pressure. This fact, in conjunction with the low absorption and small change of refractive index of CS<sub>2</sub> as a function of pressure, allowed measurements of the optical absorption edge shifts in Ge and Si to be made without having a movable sample holder in the bomb proper.

Measurements were made of the shift of the absorption edges with applied pressures ranging from atmospheric to approximately 8000 atmospheres. It was found that the Ge edge shifts to shorter wavelengths as the pressure is increased, whereas the shift in Si with increasing pressure is toward longer wavelengths. The energy shift in each case appears to be

- - - - - \*This is a coordinate program largely supported and partially staffed by Lincoln Laboratory.

\*\* In conjunction with G. B. Benedek of Group 35, Lincoln Laboratory.

\*\*\* In conjunction with D. M. Warschauer, Group 35, Lincoln Laboratory.

linear as a function of pressure; both shifts are of the order of  $10^{-12}$  ev./dyne-cm<sup>-2</sup>, but the exact values await the completion of calculations leading to iso-absorption plots yielding the change in energy versus pressure.

Tests were made on the feasibility of using Si windows for measurements at longer wavelengths than the Si cut-off. Experiment indicated that Si windows will withstand a hydrostatic pressure of 11,000 atmospheres and may possibly be arranged to go even higher, while sapphire windows (approx. 5 micron cut-off) under identical conditions withstand no more than 8 to 9000 atmospheres. Other tests indicated that stainless steel tubing 1/8" O.D. by .025" I.D. would withstand at least 18,000 atmospheres without rupture.

Using these facts, several new pieces of apparatus have been designed to enable pressures of 11 to 12,000 atmospheres to be reached with the possibility of extending the wavelength range beyond 5 microns and with much greater experimental convenience.

A new primary pressure cylinder, optical bomb, and CS<sub>2</sub> separator have been completed and other parts and accessories are being made.

### II-C Topics in the Theory of the Solid State

This program includes a number of subjects in the quantum theory of solids, many of which are closely related to the experimental program and supplement it. The various investigations include the theory of the cohesive energy and elastic constants of metals, the theory of impurity and imperfection wave functions in semiconductors and metals, and the theory of electrical resistivity as a function of pressure in metals.

#### Cohesive Energy of the Monovalent Metals

Mr. Ham has completed his thesis and is now at the University of Illinois. The work on the application of the quantum defect method to the calculation of the energy eigenvalues at symmetry points of the Brillouin zone will be continued there under the general supervision of Professor Bardeen. Before he left, Mr. Ham attempted one additional correction to the theory which was quite successful. This involved a correction for the polarization part of the potential in the outer part of the unit cell. The point is that although the electron is outside the core, the potential is still not strictly Coulombic because there is a small term which falls off as  $1/R^3$ , due to the polarizability of the ionic core. This term cannot be corrected for in the same way as the ion core potential in the method of Kambe. Instead, the procedure followed was to adjust the "observed" values of the quantum defect to new values which would have been observed had the polarization effect not been present. The correction turns out to be quite small, even for cesium, and has not yet been put into the actual cohesive energy or band energy calculations. One satisfactory result, however, is that for the higher angular momentum states in the alkalis, where there were very small quantum defect values, the polarization correction makes these go almost rigorously to zero. Actually in his calculations of the energy at symmetry points, Ham found it was necessary to assume that these higher angular

momentum quantum defects were zero; thus one conceptual difficulty in the theory has been overcome.

Progress continues on the calculation of the more extensive tables of the Coulomb wave functions. It is hoped that this project will be complete in about one month.

Dr. Kambe's work on the noble metals has been written up for publication and will be submitted shortly. Dr. Kambe has returned to Japan.

As soon as the Coulomb wave function tables are complete, the earlier calculations on the cohesive energy of the alkalis will be repeated and a final assessment of the results made. Availability from Dr. Swenson at M. I. T. of compressibility results on the alkalis at liquid helium temperature will make possible a rather exacting test of the theory. (H. Brooks)

#### Resistivity of the Alkalies

The report period has been largely occupied with checking earlier results and not much positive progress can be reported at this time. It may be stated that the theory of the resistivity of the alkali metals is still far from satisfactory. (M. Bailyn, H. Brooks)

#### Magnetic Properties of Conduction Electrons

No new developments. (H. Brooks)

#### Semiconductor Theory

The work on the investigation of a localized impurity with a narrow band gap has been completed. Efforts are now being made to carry out a similar calculation for the case where there is not only a localized potential in one cell, but also a slowly varying Coulombic potential outside the cell. The Green's function formulation extends elegantly to this case. The case of a degenerate band edge worked out previously by an approximate method, has now been treated correctly in the Green's function formulation. These treatments become identical with those of Kittel and Mitchell and of Lettinger and Kohn when equivalent approximations are made which reduce essentially to the Wannier approximation. The Green's function formulation, however, has the advantage that the variational techniques can be applied to it in a rather simple fashion.

Considerable effort has been devoted to rationalizing electrical properties of germanium and silicon in terms of recent results on the band structure resulting from cyclotron resonance experiments at Berkeley and at M. I. T. (N. Fletcher, H. Brooks)

### II-D Electron Physics

#### II-D-1 Field Emission and Work Function of Germanium, F. Allen.

Objective            To study the field emission from single germanium

crystals in a Müller-type field-emission microscope. Information on surface contaminants, work function and electron energy levels at or near the surface may be obtained, thus contributing to the understanding of semiconductor phenomena involving surfaces.

**Status** During the current period, contact potential data have been taken using the Kelvin method under a high vacuum, on a highly doped n-type single germanium crystal of approximately  $0.004 \Omega\text{-cm}$  resistivity. The crystal was so cut that the six sides provided a pair each of (100), (110) and (111) faces. The vibrating reference surface was the (110) single crystal Ni face used previously in measurements on the intrinsic Ge sample.

The heating treatments at near melting temperatures that produced significant values of c.p. for the faces of the intrinsic Ge sample failed to give any correlation between crystal direction and face c.p. for this sample. The sample was then bombarded by 500-volt argon ions and subsequently baked to about  $600^{\circ}\text{C}$ , using a technique developed at Brown University under H. E. Farnsworth. An immediate correlation of the type sought was found, and the values of c.p. for the various faces have remained essentially the same on all subsequent bombardments. The results thus far indicate that the (100) face has a work function about 0.05 volts higher than that of the (110) face, and about 0.07 volts higher than that of the (111) face. The 0.02 volt difference between the (110) and (111) faces for this sample is in the same direction as the 0.05 volt value found as the corresponding difference for the intrinsic sample but the discrepancy between the two values has yet to be explained.

The change in c.p. with illumination studied by Brattain and Bardeen has been observed on this sample at all stages of cleanliness. Values of  $(\Delta\text{c.p.})_f$  observed directly after the argon treatment, (when the surface is presumably nearly clean), are on the same order of magnitude as those resulting when the surface is highly contaminated by exposure to ozone at atmospheric pressure. A strong temperature dependence of  $(\Delta\text{c.p.})_f$  for the clean surface has been noted. For a given illumination intensity the value of  $(\Delta\text{c.p.})_f$  drops from near 100 millivolts at  $20^{\circ}\text{C}$  to about 20 mv at  $65^{\circ}\text{C}$ .

The value of the c.p. itself for this sample does not change by more than 0.1 volt from room temperature up to  $600^{\circ}\text{C}$ .

#### II-D-2 Properties of the Magnetron Diode at Microwave Frequencies, J. Bradshaw.

**Objective** To study the behavior of the space charge in a cylindrical magnetron diode and its performance at microwave frequencies. Additional theoretical and experimental work is needed to determine the space-charge distribution.

**Status** Work under this project has been terminated. Technical Reports 185 and 201, based on work done under this project, have been issued. (See Abstracts at the end of this report.)

## III

## WAVE PROPAGATION

Senior Staff: H. R. Mimno  
J. A. Pierce

Project III-1 Study of Ionospheric Abnormalities, K. Toman  
D. Davidson, J. A. Pierce, J. C. Williams.

Objective In the study of upper-atmosphere characteristics, radio reflections due to the passage of meteors, the occurrence of auroral displays, and the presence of sporadic ionization are of considerable interest.

Status E-Region

Reception on the newly assigned frequency of 3.243 Mc/s is entirely satisfactory. Interference, when present, is usually from low-powered stations, so that only the receiving location nearest the interfering source is affected.

There has been gradual increase in magnetic activity, and with this there is evidence of this there is evidence of the return of apparently moving clouds of ionization.

Auxiliary recording of 3.243 Mc/s are made at Cambridge, and these are helpful in filling in, at least qualitatively, any reception gaps in the other records. The Concord-Cambridge baseline, 27 km in length, is too short to be relied on for geometrical accuracy. The Cambridge recording employs a long sweep so that echoes of long delay and multiple-hop patterns may be examined.

F-Region

A technical report, No. 207, on this research is in press. On September 27 a new and better receiving antenna was installed for the 3.24 Mc/s receiving site in Cambridge. Current interest is directed toward analysis of the three successive magneto-ionic components appearing in the F<sub>2</sub> region at sunrise.

Project III-3 An Oscillator of Unusual Long-Term Stability, J. A. Pierce

Objective Crystal oscillators of exceptional stability provide a simple method of maintaining synchronism between a distant pulse transmitter and a local recording oscilloscope for the study of long-distance propagation. For this purpose the local oscillator is adjusted manually with as much precision as possible to the same frequency as the distant oscillator. If the oscillators are sufficiently stable, they will not

differ after a day or so by more than a part in  $10^8$ .

**Status** The improvement in behavior of our oscillators in the new constant temperature vault has been greater than we originally thought. Settling into a steady state in the new environment has demonstrably taken at least three months. The oscillator behavior is now so good that our methods of frequency measurement need improvement.

For the past two weeks, for example, the MSF standard has shown less than measurable slope ( $>1$  in  $10^{10}$ ) in 7 days and an extreme variation of  $2/10^{10}$ . In other words, the total phase drift in two weeks has been less than 50 microseconds.

Preliminary experiments with a phase-measuring servo system indicate that the MSF transmission is stable to the order of a degree of phase throughout its 1-hour duration. Standard deviations of 0.04 to 0.10  $\mu$ sec have been measured, taking the best straight line as the best curve. This indicates that the relative frequency can be measured to nearly a part in  $10^{11}$ , but the transmission stability itself has not yet been demonstrated to justify any comparisons to better than 1 in  $10^{10}$ . Further results on the measurement precision will follow improvements in the servo system.

**Project III-4 Studies of Transmission Time and Absorption in an Atmosphere of Varying Refractive Index, Mrs. E. H. Moritz, J. A. Pierce.**

**Objective** The purpose is two-fold: (1) to improve current methods of eliciting from a vertical-incidence ionospheric sounding information concerning oblique sky-wave transmission, such as maximum usable frequency, skip distance, etc., and (2) to utilize oblique-incidence relations in obtaining from the records of Project III-2 further information regarding electron distribution in the ionosphere and its effect on transmission time and sky-wave signal strength.

**Status** The reduction of data on the noise survey continues to fully occupy Mrs. Moritz, as the extreme interest in other activities (see III-5 below) has denied her much help. The work is drawing to an orderly conclusion, however.

**Project III-5 Carrier-Frequency Phase Studies, J. A. Pierce, C.K. H. Tsao**

**Objective** To exploit some of the possibilities of coherent detection at low radio frequencies. By utilizing the inherent phase stability of transmission at these frequencies, field strength can be measured well below the ambient noise level and new data on transmission time can be collected.

**Status** The settling down of our oscillators and minor improvements in other respects have resulted in startling improvements in the GBR records. When the signal is transmitted for most of

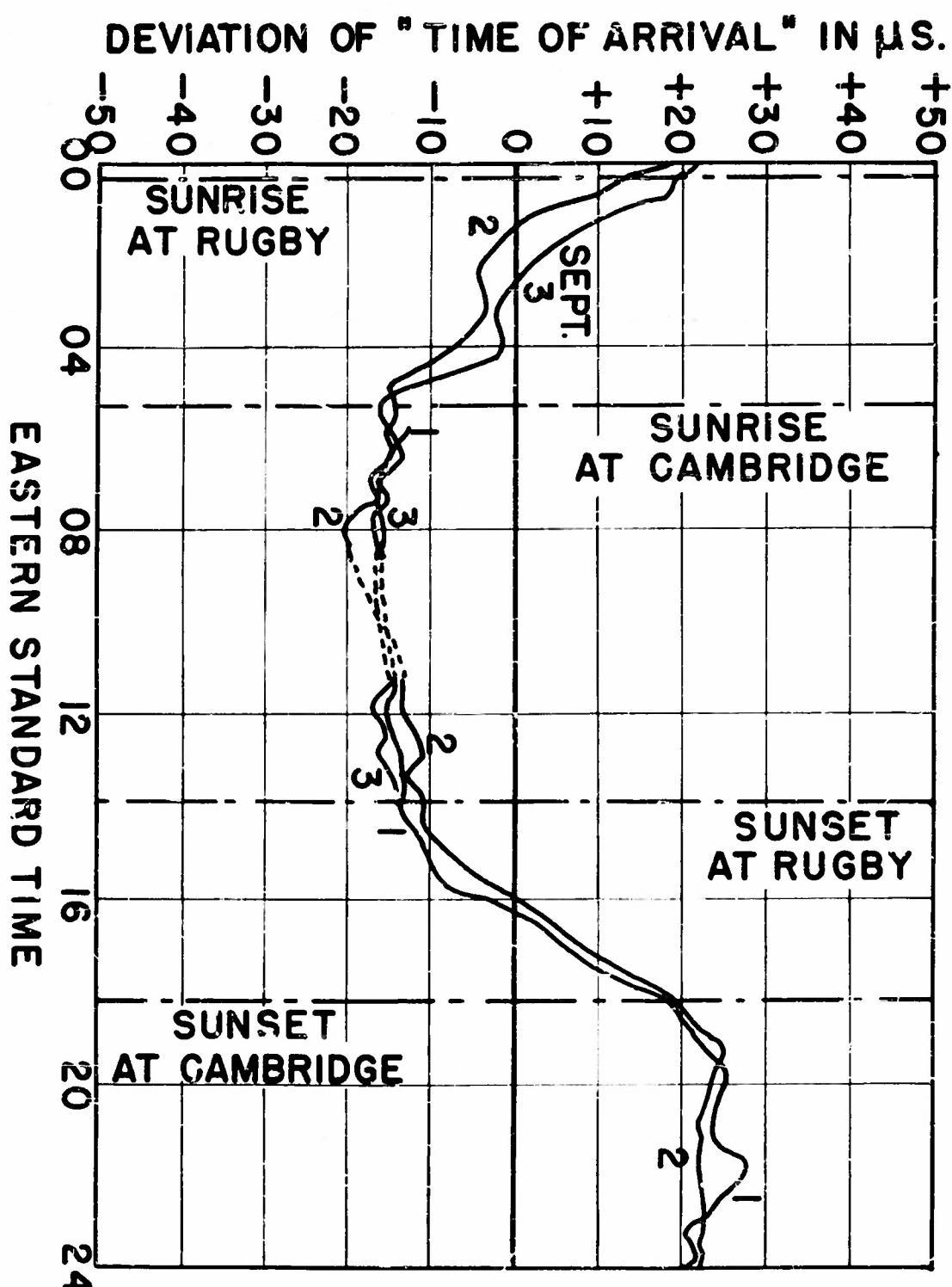


FIG.III - I THE DIURNAL VARIATIONS OF  
GBR'S TRANSMISSION TIME  
FOR SEPTEMBER 1 TO 3, 1954

the time over two or three days (a relatively rare circumstance) excellent diurnal variations are being obtained.

Even if our oscillator frequency is not exact, in terms of GBR, its value can be determined in retrospect to better than 1 part in  $10^{10}$ . The best example to date, when all was running well and GBR was transmitting, is given in Fig. III-1. Here the oscillator phase variations have been removed and all that remains is the true diurnal variation of the transmission time with minor fluctuations that may be either real or instrumental. It will be observed that the repetition from day to day is excellent, indicating that Doppler effects are minor (in terms of day-to-day consistency). The times at which the day-to-night transitions begin and end are very reasonable.

This work has several important conclusions, the simplest being that international frequency comparison to very great precision is easily possible, and that a new propagational tool has been found. Others are the implications for aids to navigation and for narrow-banding to provide slow communication of very high reliability. The utility of a VLF standard frequency transmission becomes clearer and clearer.

A technical report, TR 209, on these matters has been prepared and is now in press. It gives the background data requisite for appreciation of Fig. III-1 and develops the conclusions in greater detail than is done above.

Abstracts of Technical Reports

(Completed since July 1, 1954)

Parasitic Sleeve Antennas

C. E. Falick

Technical Report No. 157

This research treats theoretically and experimentally the problem of the parasitic sleeve antenna, consisting of a coaxial sleeve mounted on a cylindrical antenna center-driven over an image plane. It is first shown that the problem of the parasitic sleeve antenna can be resolved into two problems. (1) The center-driven cylindrical antenna of discontinuous thickness, (2) the sleeve antenna of discontinuous radius.

Equipment is described for measuring both the input impedance and the current distribution of a parasitic sleeve antenna. Current distributions on the outside of the sleeve are measured by a separate probe connected to the detecting system by miniature cable coiled inside the sleeve wall and threaded down the inside of the antenna. The errors in impedance measurement are discussed and it is shown that the measurements are accurate to within a one per cent circle on a Smith Chart. Errors in phase measurements using a slotted matched line are considered and a method of minimizing errors due to a poorly matched line is presented for the case when it is impossible to adjust the relative amplitude of the two comparison signals.

Graphs of input impedance vs sleeve-tuning of an internal sleeve antenna in which case the sleeve is contained within the cylindrical antenna are shown for several positions of the sleeve mouth. As predicted, the locus is a circle on a Smith Chart. Curves of the measured input impedance of a large number of parasitic sleeve antennas are given for various antenna lengths, sleeve positions, sleeve lengths, and sleeve diameters. Several representative current distributions are shown and the general behavior of the current is shown succinctly in a novel presentation. Finally, the effectiveness of the parasitic sleeve antenna in reversing the current in the outer portion of the antenna is compared with that of the phase-reversing stub.

Antennas of Discontinuous Radius

C. E. Falick

Technical Report No. 171

This research treats theoretically and experimentally the problem of the antenna with discontinuous radius.

A set of three simultaneous integral equations with appropriate boundary

boundary conditions is written for the special case of two discontinuities in radius which set corresponds to the single Hallén equation for the uniform cylindrical antenna. A corresponding set of equations for the single discontinuity has been derived previously by Uda and Mushiake but was not evaluated except for the special case when the length of the antenna was near one quarter-wavelength. Approximations to the solution of the integral equations are then considered. A "transmission-line analogy" method is described which utilizes the known input impedance of the uniform antenna and the analogy between an antenna and a corresponding transmission line to approximate the input impedance and current distribution of an antenna with discontinuous radius. A comparison with the experimental values shows that this method is more accurate than that of Uda and Mushiake, and that for sufficiently thin antennas the results are acceptable for most engineering purposes. The transmission-line analogy method is then modified by using a corrected current distribution obtained from a first-order iteration of the zero-order current in each section of the antenna.

The measured input impedance of several antennas having a single discontinuity in radius is presented. It is noted that input impedance is almost independent of the thickness of the outer section if the length of the outer section is one quarter-wavelength. Measurements for a case calculated by Uda and Mushiake are included for comparison.

The measured input impedance of several closed sleeve antennas is presented as a function of the position of the thicker section or closed sleeve along the antenna. The effect of such a section is large and is not associated with the increased length of the current path. Corresponding calculated values using the transmission-line analogy method, the first-order correction theory, and Schelkunoff's non-uniform transmission-line theory are included to show the limits of validity of the various approximations. The current distributions along some of the closed sleeve antennas are also shown.

Finally, a comparison is made of the several approximate methods of calculation and the effect of neglecting the coupling between sections and the capacity effect at the junction regions is discussed.

#### A Probe Signal Study of the Hull Magnetron Diode

John A. Bradshaw

Technical Report No. 185

The Hull magnetron diode is a vacuum tube essentially consisting of two concentric metal cylinders. In the diode used to obtain the results herein reported, the inner cylinder was a nickel sleeve  $1/4"$  in diameter and it carried an oxide coating 10 cm long. The outer cylinder was a heavy copper anode. The sleeve could be heated above  $1200^{\circ}\text{K}$  and a steady voltage could be applied between it and the anode.

Anode and sleeve were also sections integrated in a coaxial trans-

mission line between a high-frequency oscillator and a detector. A TEM wave could then pass from the oscillator through the diode to the detector, probing or exciting, as it passed, the space charge of electrons emitted from the hot oxide coating. When a uniform steady magnetic field is imposed on the diode, parallel to its cylindrical axis, the flow of electrons from sleeve to anode may be virtually cut off, and the diode operates as a magnetron.

The probe signal suffers a sharp resonance absorption in transmission through the magnetron when the cyclotron frequency (characteristic of an electron in the magnetic field) lies close to the probe frequency. Near this absorption frequency the transmitted signal suffers changes of phase as well as of amplitude. These were observed over wide ranges of probe frequency, anode voltage, magnetic field and cathode temperature. The changes were correlated with other effects, the disturbance of the residual current by the probe signal, the noise generated in the tube, the signal at the second harmonic of the probe frequency, also generated in the tube and so on. The second half of this report outlines these changes and related effects.

For interpreting these phenomena, the basis is Hull's set of equations of the electronic orbit. In the first half of this report, an examination of the orbit equations yields an expression for the conductivity of the space-charge cloud. The conductivity depends on the ratios of probe and orbit frequency to the cyclotron frequency,  $u$  and  $u_0$ , respectively, on a phenomenological damping parameter  $\gamma$ , and on the average charge density. This density in turn depends on the ratio of anode voltage to the square of the magnetic field.

The probe signal measurements imply a particular distribution of the space charge within the magnetron diode. The features of this distribution form the subject of a companion report, TR 201.

### The Distribution of Space Charge in the Hull Magnetron Diode

John A. Bradshaw

Technical Report No. 201

If, in a magnetron diode, the current is not limited by cathode temperature, then the radial dependence of the potential and the charge density functions may be found before cutoff from Langmuir's solution, modified by effects of increasing transit time. In the cutoff transition and after cutoff, the cathode temperature, as reflected in the distribution of electronic velocities at emission affects the potential and density functions. Data on anode current characteristics are compared with calculations based on models that include these temperature effects.

The data from TR 185 on probe signal absorption are interpreted as showing that the space-charge cloud extends clear to the anode even after cutoff, and is not bounded by the so-called Hull radius. This and other evidence points to electronic interactions as the source of a

mechanism for transporting charge through the cloud from cathode to anode. Finally, the electronic distribution in phase space is stated in a form which could be verified and refined by further experiments.

**A Theoretical and Experimental Investigation of  
the Radiation of a Vertical Antenna over a Coated Conductor**

**Donald B. Brick**

**Technical Report No. 195**

The idealized problem of a base-driven cylindrical antenna over a perfectly conducting dielectric-coated image plane is treated theoretically. An integral equation which is utilized in arriving at an approximation to the current distribution on the antenna is derived. Use is made of the results of a previous report in the derivation.

The far-zone fields for the configuration are shown to be composed of two types of waves - those of spherical type, the radiation or compensating field, and those of cylindrical type which attenuate exponentially with height above the dielectric surface, the surface or guided waves. These two are combined at a typical distance to yield theoretical field patterns.

The experimental approximation to the idealized configuration is described at length. Measurements made using this apparatus are seen to be in good agreement with theory, thus verifying the theory contained in this and the previous report. Experimental results are also given for partially coated surfaces as an aid towards the understanding of the phenomena.

Considerable space is devoted to the choice and the calibration of the measuring antennas. In the course of this study a new concept of the complex effective receiving length is introduced.

**Experimental Study of Collinear Slot Antenna  
(An Application of Babinet's Principle)**

**T. Kaliszewski**

**Technical Report No. 202**

The effect of a high-impedance transmission-line coupling on the current and phase relationship on a three-element, center-driven, collinear slot array at 10 cm wavelength is studied. A brief summary of theoretical results is followed by a detailed description of apparatus and the measuring procedures. Use is made of Babinet's principle in carrying out the measurements in an attempt to assess the validity of complementary slot techniques.

Results thus obtained are compared with the theory and with alternative measurements made on a similar structure employed at much lower frequency.

**Movement of the F-region****Kurt Toman****Technical Report No. 207**

In the course of a fixed-frequency ionospheric study, employing a pulse-triggered transmitter operating on 3.5 Mc/s and three spaced-receivers, the transmission-delay was continuously recorded. Aside from a vertical-incidence transmission two oblique transmissions were thus available with 62 and 109 kilometers as baselines, the latter being correspondingly oriented in an approximate west-east and north west-south east direction.

An analysis of the echoes from the F-region was made for the period between August 1952 and December 1953. Successive irregularities observed simultaneously on three records displayed frequently consistent time displacements. Assuming the mid-points of the transmissions to be characteristic and preferred areas for the reflection of the h. f. -pulses, the time-displacements were interpreted as being due to a mechanical motion of the F-region. Direction and speed of this movement was thus obtained and semiannual and annual periods of these components became apparent.

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